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## OCA PAD AMENDMENT - PROJECT HEADER INFORMATION

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Center shr #:

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Document : GRANT  
Contract entity: GTRC

Contract#: NAG3-1455  
Prime #:

Mod #: UNILATERAL NCE

Subprojects ? : N  
Main project #:

CFDA: 43.002  
PE #:

Project unit:  
Project director(s):  
SMITH M K

MECH ENGR  
MECH ENGR

Unit code: 02.010.126  
(404)894-3826

Sponsor/division names: NASA  
Sponsor/division codes: 105

/ LEWIS RESEARCH CTR, OH  
/ 011

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Contract value	0.00	209,999.00
Funded	0.00	209,999.00
Cost sharing amount		0.00

Does subcontracting plan apply ? : N

Title: THE BEHAVIOR OF UNSTEADY THERMOCAPILLARY FLOWS

## PROJECT ADMINISTRATION DATA

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Security class (U,C,S,TS) : U

ONR resident rep. is ACO (Y/N): Y

Defense priority rating :

supplemental sheet

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GIT X

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Administrative comments -

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Closeout Notice Date 11-NOV-1997

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Project Number E-25-X77

Center Number 10/24-6-R7735-0A0

Project Director SMITH, MARC

Project Unit MECH ENGR

Sponsor NASA/LEWIS RESEARCH CTR, OH

Division Id 3391

Contract Number NAG3-1455

Contract Entity GTRC

Prime Contract Number

Title THE BEHAVIOR OF UNSTEADY THERMOCAPILLARY FLOWS

Effective Completion Date 23-MAY-1997 (Performance) 23-AUG-1997 (Reports)

Closeout Action:	Y/N	Date Submitted
Final Invoice or Copy of Final Invoice	Y	
Final Report of Inventions and/or Subcontracts	Y	
Government Property Inventory and Related Certificate	Y	
Classified Material Certificate	N	
Release and Assignment	N	
Other	N	
Comments		

## Distribution Required:

Project Director/Principal Investigator	Y
Research Administrative Network	Y
Accounting	Y
Research Security Department	N
Reports Coordinator	Y
Research Property Team	Y
Supply Services Department/Procurement	Y
Georgia Tech Research Corporation	Y
Project File	Y

NOTE: Final Patent Questionnaire sent to PDPI



E-25-X77  
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# The Behavior of Unsteady Thermocapillary Flows

Marc K. Smith and Jean N. Koster

## NAG3-1455 – Semi-Annual Status Report –Year 1

### *Theoretical Work*

The theoretical portion of this project has been concentrating on the description of the thermocapillary instability of a thin liquid layer contained in a rectangular enclosure. The assumption that the average thickness of the liquid layer  $d$  is smaller than the streamwise length scale  $L_x$  or the spanwise length scale  $L_y$  introduces two small parameters called aspect ratios into the description of this flow. A singular perturbation analysis for small values of the aspect ratios allows us to compute the outer solution, which is the flow field away from the sides of the enclosure. The computed velocity field is known in terms of the layer thickness. Then using the kinematic condition on the free surface, we have derived an evolution equation for the shape of the free surface. This is a nonlinear partial differential equation in terms of time and the streamwise and spanwise directions.

To solve the evolution equation, we need boundary conditions along the container walls. These are found by looking at the inner boundary layer problems for the flow near the container walls. Without having to solve for these flows explicitly, we can develop effective boundary conditions for the layer thickness at these walls. The result is a well-posed evolution equation for the shape of the free surface of the liquid layer.

Our analysis of this equation started with finding the steady-state shape for a two-dimensional container. The evolution equation reduces to the one found by Sen and Davis (1982). The equation is solved numerically using a spectral method based on a Chebyshev polynomial expansion for the free-surface position. The nonlinearity is dealt with using a Newton-Kantorovich iteration scheme described by Boyd (1989). The numerical solution agrees very well with the exact solution that can be found for this special case.

The next task was to develop the methodology to determine the stability of this steady-state flow. A linear stability analysis was done to form an eigenvalue problem that was solved numerically. The added difficulty here was that the basic state is only known numerically. The result for the two-dimensional container was complete stability as expected.

Presently, we are considering a three-dimensional container. The evolution equation has been extended to the spanwise direction. The effective boundary conditions for the free surface on the sidewalls are being derived. Once these are obtained, the basic state and stability will be found as before.

Future work will include considering the geometry of a thin annulus, which is the three-dimensional rectangular container wrapped around once. This will mimic the flow in a float-zone, which is a geometry of some commercial interest. Then we can also consider the flow in a long narrow cylinder itself. Finally, we plan to include the effects of inertia to leading order in the evolution equation for the free surface. This can be done by using a Karman-Pohlhausen method to approximate the streamwise velocity profile.

### *Experimental Work*

The experimental work has been conducted at the University of Colorado at Boulder under the direction of Prof. Jean N. Koster. The goal for this first year is to set-up and begin the experiments on transparent liquids using optical techniques. So far, the optical test cell has been designed and constructed. It is an open, rectangular Plexiglas container about five inches long and half an inch wide. The two ends of the cell are held at different fixed temperatures, while the lower surface, which is Plexiglas, simulates an insulated surface. The container will be filled with a layer of silicone oil to a depth of at most 1 cm. This oil will be driven in a thermocapillary motion by the temperature difference of the ends of the cell.

The bulk of the money this first year was used to help purchase a laser doppler velocimeter. This device has been installed in Prof. Koster's laboratory and is being readied for use.

Another aspect of the experiments is the visualization of the free-surface deflection. The technique that is used is to record the reflection of an incident laser beam on the free surface. As the free surface deflects, the reflected beam moves. The motion of this beam can be directly related to the free surface deflection and orientation. The current technique is able to make a video record of the beam motion during the course of the experiment.

Measurements in the test cell will begin once a graduate student from Germany arrives at Boulder and gets up to speed on the experiment and the available facilities.

Once these measurements are done, we will begin the measurements of the velocity field in a liquid metal, probably gallium, using the x-ray flow visualization technique.



### *References*

Boyd, J. P. 1989 *Chebyshev & Fourier Spectral Methods*. In Lecture Notes in Engineering, Vol. 49 (Springer-Verlag, Berlin, Heidelberg).

Sen, A. K. & Davis, S. H. 1982 Steady thermocapillary flows in two-dimensional slots. *J. Fluid Mech.* **121**, 163-186.

### *Bibliography*

No published work on this problem has appeared at this time.

### *Students*

David R. Vrane - Georgia Tech

The Behavior of Unsteady Thermocapillary Flows  
Marc K. Smith

Project Number NAG3-1455  
Progress Report — October 31, 1996

Steady, two-dimensional flow states in a thin liquid layer contained in a finite rectangular cavity have been found in terms of the free-surface deformation over a finite range of capillary numbers. This range is bounded from above by a limit point value of the capillary number. Below this value, two solution branches occur: a high-deformation branch; and a low-deformation branch. The high-deformation solution branch is unstable and the layer tends toward either the stable, lower-branch solution, or to film rupture. Above the limit-point value, no steady-state flows exist. The layer seems to progress towards a ruptured-film state at the hot end of the film.

The influence of inertia in the liquid layer was examined by allowing for a flow with a large Reynolds number. The resulting leading-order momentum equation was solved approximately using an integral technique. A parabolic longitudinal velocity was assumed, it was substituted into the momentum equation, and the result was averaged over the thickness of the layer. The resulting momentum equation, together with a mass conservation equation was solved using the same pseudo-spectral techniques used for the viscous-dominated case. The results show that the effect of inertia in the liquid is to reduce the free-surface deformation for a given capillary number and to increase the value of the limit point capillary number. The upper-branch solution is still unstable and the lower-branch solution is stable. However, at low capillary numbers, the preferred disturbance mode for the lower branch is oscillatory when inertia is included. We have shown that for small enough capillary numbers, the free surface behaves like a damped vibrating beam.

Steady, axisymmetric flow states have also been found in terms of free-surface deformation over a finite range of capillary numbers for flows occurring in two additional geometries: a wide, flat annular cavity with a flat free-surface; and a shallow cylindrical cavity with a cylindrical free-surface. The basic flow states in the annular cavity are similar to the rectangular cavity discussed earlier. The effect of azimuthal curvature is to destabilize the film when the inner radius of the film is large. This means that the limit-point capillary number is decreased. However, when the inner radius is small enough, the film is stabilized significantly. Near the point where the inner radius goes to zero, the film is completely stabilized. In this situation, a steady-state film profile is possible for all values of the capillary number. The limit-point capillary number goes off to infinity.

In a shallow cylindrical cavity, the inherent cylindrical curvature of the free surface always has a destabilizing effect on the film flow. The extra destabilization is due to a Rayleigh capillary instability associated with this geometry.

Our future work will be to include the effect of liquid inertia in the calculations for the cylindrical cavity. The stabilization effect of inertia may offset the destabilization effect inherent in the cylindrical geometry and result in a stable film flow on the cylinder. This type of behavior could be very significant in terms of understanding the stability of thermocapillary flow in liquid bridges and in the float-zone crystal growth process.



# **The Behavior of Unsteady Thermocapillary Flows**

Marc K. Smith

Project Number NAG3-1455  
Final Research Report — August 23, 1997

Surface-tension-driven flows are of central concern to the NASA microgravity fluid dynamics program because they can be the dominant flows in many processes used in a microgravity environment. This project has been an investigation of the behavior of the thermocapillary flow of a thin liquid layer contained in three different geometrical containers: a rectangular cavity, a flat annular cavity, and a cylindrical cavity. In this work, we used asymptotic methods to derive nonlinear evolution equations and boundary conditions for the thermocapillary flows in these geometries. From the numerical solution of these equations, we determined the possible flows, analyzed their stability, and studied the effect of the various system parameters, the effect of interfacial deformation, and the effect of the ends on the flow.

## **Summary of Results**

Steady, two-dimensional flow states in a thin liquid layer contained in a finite rectangular cavity have been found in terms of the free-surface deformation over a finite range of capillary numbers. This range is bounded from above by a limit point value of the capillary number. Below this value, two solution branches occur: a high-deformation branch; and a low-deformation branch. The high-deformation solution branch is unstable and the layer tends toward either the stable, lower-branch solution, or to film rupture. Above the limit-point value, no steady-state flows exist. The layer seems to progress towards a ruptured-film state at the hot end of the film.

The influence of inertia in the liquid layer was examined by allowing for a flow with a large Reynolds number. The resulting leading-order momentum equation was solved approximately using an integral technique. A parabolic longitudinal velocity was assumed, it was substituted into the momentum equation, and the result was averaged over the thickness of the layer. The resulting momentum equation, together with a mass conservation equation was solved using the same pseudo-spectral techniques used for the viscous-dominated case. The results show that the effect of inertia in the liquid is to reduce the free-surface deformation for a given capillary number and to increase the value of the limit-point capillary number. The upper-branch solution is still unstable and the lower-branch solution is stable. However, at low capillary numbers, the preferred disturbance mode for the lower branch is oscillatory when inertia is included. We have shown that for small enough capillary numbers, the free surface behaves like a damped vibrating beam.

Steady, axisymmetric flow states have also been found in terms of free-surface deformation over a finite range of capillary numbers for flows occurring in a wide, flat

annular cavity with a flat free-surface, and in a shallow cylindrical cavity with a cylindrical free-surface. The basic flow states in the annular cavity are similar to the rectangular cavity discussed earlier. The effect of azimuthal curvature is to destabilize the film when the inner radius of the film is large. This means that the limit-point capillary number is decreased. However, when the inner radius is small enough, the film is stabilized significantly. Near the point where the inner radius goes to zero, the film is completely stabilized. In this situation, a steady-state film profile is possible for all values of the capillary number. The limit-point capillary number goes off to infinity.

In a shallow cylindrical cavity, the inherent cylindrical curvature of the free surface always has a destabilizing effect on the film flow. The extra destabilization is due to a Rayleigh capillary instability associated with this geometry.

## **Conclusions**

We have found that it is possible to have steady thermocapillary flows in any of these three systems provided the liquid layer is not too thin or the surface tension is not too small. If these conditions are not met, then no steady solutions are possible and the film thins and approaches a condition of rupture near the hot end.

Future work on these problems will include the effect of liquid inertia in the calculations for the cylindrical cavity. The stabilization effect of inertia may offset the destabilization effect inherent in the cylindrical geometry and result in a stable film flow on the cylinder. This type of behavior could be very significant in terms of understanding the stability of thermocapillary flow in liquid bridges and in the float-zone crystal growth process.

## **Publications and Presentations**

### **Refereed Publications:**

Vrane, D. R. & Smith, M. K., "Free-surface instabilities in confined, low Prandtl number, thermocapillary-driven flows. Part 1. Viscous and inertial effects," submitted to JFM.

Vrane, D. R. & Smith, M. K., "Free-surface instabilities in confined, low Prandtl number, thermocapillary-driven flows. Part 2. The influence of domain curvature," submitted to JFM.

### **Conference Presentations:**

Vrane, D. R. & Smith, M. K., "The Behavior of Unsteady Thermocapillary Flows," Second Microgravity Fluid Physics Conference, Cleveland, OH, June 21-23, 1994.

Vrane, D. R. & Smith, M. K., "The Influence of Domain Curvature on the Stability of Viscously-Dominated Thermocapillary Flows," AMS-IMS-SIAM Joint Summer Research Conference on 'Analysis of Multi-Fluid Flows and Interfacial Instabilities', Seattle, WA, July 23-27, 1995.



Vrane, D. R. & Smith, M. K., "The High Capillary Number Behavior of a Viscously-Dominated, Thermocapillary-Driven Flow in a Two-Dimensional Rectangular Cavity," Forty-Seventh Meeting of the American Physical Society - Division of Fluid Dynamics, Atlanta, GA, November 20-22, 1994.

Vrane, D. R. & Smith, M. K., "The Influence of Domain Curvature on the Stability of Viscously-Dominated Thermocapillary Flows," Forty-Eighth Meeting of the American Physical Society - Division of Fluid Dynamics, Irvine, CA, November 19-21, 1995.

Vrane, D. R. & Smith, M. K., "Inertial Stabilization of Capillary Break-up in a Thermocapillary-Driven Cylindrical Cavity," Forty-Ninth Meeting of the American Physical Society - Division of Fluid Dynamics, Syracuse, NY, November 24-26, 1996.

Dissertation:

Vrane, D. R., "The stability of thermocapillary-driven flows in finite regimes," Georgia Institute of Technology, 1996.

Invited Seminars:

Smith, M. K., "Confined Liquid Films Driven by Surface-Tension Gradients," Department of Chemical Engineering, University of Notre Dame, South Bend, IN, October 25, 1996.